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Optimization of transmission capacities for multinodal markets

Alexander Vasin[a,1], Marina Dolmatova[a]

[a]*Faculty of Computational Mathematics and Cybernetics, Lomonosov Moscow State University, GSP-1, Leninskie Gory, 119991 Moscow, Russia*

Abstract

This paper considers a homogenous good competitive market consisting of *n* local markets with given supply and demand functions. The markets are connected by several transmission lines. For every line, the cost functions of transmission capacity increment include fixed and variable components. We set a problem of the total social welfare optimization and discuss its generalization for markets with exporting and importing nodes. We distinguish several cases where the structure of connections corresponds to a tree-type graph and the social welfare function is submodular or supermodular with respect to the set of expanded transmission lines. These properties permit to employ known efficient algorithms that determine the optimal transmission capacities.

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Keywords: transmission network, transmission expansion, total welfare, homogeneous good

1. Introduction

Markets of natural gas, oil, electricity and other resources play an important role in economies of many countries. An essential component of such markets is a transmission system. Consumers and producers are located at different nodes, and transmission capacities of the lines between the local markets are limited. By recent estimates, the transmission costs may exceed 50% of the electricity price for the industry consumers in Russia. Therefore, the problem of transmission system optimization is of practical interest.

The previous researches on such markets [2, 3, 7, 10, 12, 13] consider primarily models with a fixed network structure. Wilson [14] analyzes market architecture issues for electricity industry. The recent paper [4] determines the optimal transmission capacity of one line for a two-node market, taking into account the transmission losses and the costs of transmission line construction. The present study aims to generalize these results for markets with several transmission lines, where the structure of connections corresponds to a tree-type graph. We consider the total welfare optimization problem with account of the production costs, consumers' utilities and the costs of transmission lines expansion. In our model, demand functions reflect possibilities for the welfare increase due to the reduction of the energy prices at local markets. The difficulty of the problem under consideration is that an expansion of any line requires valuable fixed costs. If the optimal set of expanded lines was known, the problem would be convex and we could solve it by standard tools. However, the efficient search of this set under a large

*Corresponding author. Tel.: +7-495-939-2491 ; fax: +7-495-939-2596 .
E-mail address: foravas@yandex.ru.

number of connecting lines requires special methods. In general the problem is NP-hard [6]. We distinguish several cases where the social welfare function is submodular or supermodular with respect to the set of expanded transmission lines. So the known methods of successive calculations and excluding rules are applicable ([1], [9]).

Below we assume that the transmission system works as if there is a perfect competition among intermediaries who can buy a good at one node and sell it at the other. For many real markets, for instance, electricity markets, the system operators regulate the flows according to this assumption (see [7]). [8] studies the different case and provides a literature review on transmission grid expansion for electricity markets with transmission lines controlled by private transmission companies.

2. Formal model of the market

We consider a homogeneous good market consisting of several local markets and a network transmission system. Let N denote the set of nodes and $L \subseteq N \times N$ be the set of edges.

Every node $i \in N$ corresponds to a local perfectly competitive market. Demand function $D_i(p)$ and supply function $S_i(p)$ characterize respectively consumers and producers in the market and meet the standard conditions: $D_i(p)$ is continuous and monotonically decreasing in p, where $D_i(p) > 0$, and $D_i(p) \to 0$ as $p \to \infty$; $S_i(0) = 0$ and $S_i(p)$ is non-decreasing in p. The demand function relates to the consumption utility function: $U_i(q) = \int_0^q D^{-1}(v)\,dv$. The supply function $S_i(p)$ determines the optimal production volume at the node i for the profit maximization problem under a given price : $S_i(p) = Arg\max_v(pv - c_i(v))$, where $c_i(v)$ is the minimal production cost of volume v at node i. Note that the total profit of producers at node i under price $\overline{p}$ is $Pr_i(\overline{p}) = \int_0^{\overline{p}} S_i(p)dp$.

For any $(i, j) \in L$, the edge (i, j) represents the transmission line connecting local markets i and j. The line is characterized by the initial transmission capacity Q^0_{ij}, the unit transmission cost e^{ij}_t, the cost function E_{ij} of the transmission capacity increment, including fixed costs e^{ij}_f and variable costs $e^{ij}_v(Q_{ij}, Q^0_{ij})$. For any $(i, j) \in L$, let q_{ij} denote the flow from the market i to market j, $q_{ij} = -q_{ji}$.

Thus, the total transmission costs for edge (i, j) are:

$$E_{ij}\left(q_{ij}\right) = \begin{cases} e^{ij}_f + e^{ij}_v\left(\left|q_{ij}\right|, Q^0_{ij}\right) + e^{ij}_t\left|q_{ij}\right|, & \text{if } \left|q_{ij}\right| > Q^0_{ij}, \\ e^{ij}_t\left|q_{ij}\right|, & \text{if } \left|q_{ij}\right| \le Q^0_{ij}. \end{cases} \tag{1}$$

In this paper we consider the case where the costs do not depend on the direction of the flow and the flows do not change in time. Then the final transmission capacity Q_{ij} is $|q_{ij}|$, if $\left|q_{ij}\right| > Q^0_{ij}$, otherwise it is Q^0_{ij}. The cost of the line expansion $e^{ij}(Q_{ij}) = e^{ij}_f + e^{ij}_v(Q_{ij}, Q^0_{ij})$ is the overnight construction cost amortized over the life-time T_{ij} of the line using discount rate r: $e^{ij} = r\frac{OC_{ij}}{1-e^{rT_{ij}}}$ (see [11] for the detailed discussion).

We distinguish fixed component e^{ij}_f, assuming that $e^{ij}_v\left(Q^0_{ij}, Q^0_{ij}\right) = 0$ and e^{ij}_v is a monotonous convex function of increment $(Q_{ij} - Q^0_{ij})$. In many practical problems the fixed components are rather substantial, and this determines the complexity of the optimization problem considered below.

Consider the incidence matrix $A = \left\{a_{ij}\right\}$, $i \in N$, $j \in N$ corresponding to the graph G: $a_{ij} = 1$, if $(i, j) \in L$, otherwise $a_{ij} = 0$. Denote $Z(i) = \{j | a_{ij} = 1\}$ the set of nodes connected with node i.

Under any fixed flows of the good $\overrightarrow{q} = \left(q_{ij}, (i, j) \in L\right)$ and production volumes $\overrightarrow{v} = (v_i,\ i \in N)$, the consumption volumes $(\widehat{v_i},\ i \in N)$ are obtained from the balance equations:

$$\widehat{v_i} = v_i + \sum_{j \in Z(i)} q_{ji},\ i \in N.$$

The total social welfare for the network market is the total consumption utility over the market minus the total production and transmission costs:

$$\overline{W}(\overrightarrow{q}, \overrightarrow{v}) = \sum_{i \in N} [U_i\left(v_i + \sum_{l \in Z(i)} q_{li}\right) - c_i(v_i)] - \sum_{(i,j) \in L,\quad i<j} E_{ij}\left(q_{ij}\right). \tag{2}$$

An alternative representation of this value is the total profit of all the agents in the market: producers, consumers and the transmission system. Indeed, under strategies $\overrightarrow{q}$, $\overrightarrow{v}$, the price $p_i(\overrightarrow{q}, \overrightarrow{v})$ at node i meets the balance equation: $D_i(p_i) = v_i + \sum_{j\in Z(i)} q_{ji},\ i \in N$. The producers' profit is $Pr_i = p_i v_i - C_i(v_i)$, the consumers surplus is $CS_i = \int_{p_i}^{\infty} D_i(p)dp$, and the benefit of the transmission system is determined as $T\left(\overrightarrow{p}, \overrightarrow{q}\right) = \sum_{(i,j)\in L,\ i<j} [\left(p_j\left(\overrightarrow{q}, \overrightarrow{v}\right) - p_i\left(\overrightarrow{q}, \overrightarrow{v}\right)\right) q_{ij} - E_{ij}(q_{ij})]$. Then $\overline{W}\left(\overrightarrow{q}, \overrightarrow{v}\right) = \sum_{i\in N}(Pr_i(\overrightarrow{q}, \overrightarrow{v}) + CS_i(\overrightarrow{q}, \overrightarrow{v})) + T\left(\overrightarrow{p}, \overrightarrow{q}\right)$. The welfare optimization problem under consideration is

$$\max_{\overrightarrow{q}, \overrightarrow{v}} \ \overline{W}(\overrightarrow{q}, \overrightarrow{v}) \,. \tag{3}$$

The next proposition solves "a half of the problem". Let $\triangle S_i(p_i) = S_i(p_i) - D_i(p_i)$ denote the supply-demand balance at the node i under the given price.

Theorem 2.1. *Under any fixed flows of the good between the local markets* $\left(q_{ij},\ (i,j) \in L\right)$ *, the optimal production volume at node i is* $v_i = S_i(\widetilde{p_i})$*, where* $\widetilde{p_i}$ *meets equation* $\triangle S_i(\widetilde{p_i}) = \sum_{j\in Z(i)} q_{ij}$

Proof. Proceeding from representation (2) of the total welfare, the problem (3) under a fixed $\overrightarrow{q}$ is reduced to independent optimization problems:

$$v_i^* = arg\max_{v_i}\left(U_i\left(v_i + \sum_{l\in Z(i)} q_{li}\right) - c_i(v_i)\right),\ i \in N.$$

According to the First Welfare Theorem [5], the solution for market i corresponds to the Walrasian equilibrium of the market under the given flows. □

Consider a modification of the total welfare concept for the case where some final nodes are either exporting or importing the good. At such nodes, agents from the market under consideration interact with outsiders. At an exporting node, there is no production, and the transmitted good is sold to foreign consumers. The node i is characterized by the demand function $D_i(p_i)$ for the exported good, and the total social welfare component for this node is $W^i(q_{\sigma(i)i}) = q_{\sigma(i)i} \cdot D_i^{-1}(q_{\sigma(i)i}) = D_i(p_i) \cdot p_i$, where $\sigma(i)$ is a the preceding node for the node $i \in N$. So $W^i(q_{\sigma(i)i})$ is the income from the sales. At the importing node, the transmission system buys the good from foreign producers. Such node i is characterized by supply function $S_i(p_i)$, and the input to the social welfare is calculated as $W^i(p_i) = -q_{i\sigma(i)} S_i^{-1}(q_{i\sigma(i)}) = -S_i(p_i) \cdot p_i$.

Similar modifications should be specified for foreign companies operating as producers or consumers in the national market. Note that in context of the national social welfare optimization not formal registration of a company but actual distribution of its profit and investments should determine its specification as foreign.

3. Properties of the welfare function for multi-node network markets

Let us remind the definitions [9] of submodular and supermodular functions and some of their properties, used for maximization of these functions. A function $\widetilde{W}(\omega)$ defined for each subset $\omega \in \bar{L}$ of a finite set $\bar{L}$, is *submodular* (respectively, *supermodular*) if for every $L', L'' \subseteq \bar{L}$ it holds that

$$\widetilde{W}(L') + \widetilde{W}(L'') \geq \ (respectively, \leq)\ \widetilde{W}(L' \cup L'') + \widetilde{W}(L' \cap L'').$$

A submodular function $\widetilde{W}(L)$ meets the following properties.

1. Let $\widetilde{W}(L) \geq \widetilde{W}(L \setminus \{i\})\ \forall i \in L$. Then $\max_{S\subseteq L} \widetilde{W}(S) = \widetilde{W}(L)$.
2. Let $\widetilde{W}(i) < \widetilde{W}(\emptyset)$. Then $i \notin S^* := Argmax_{S\subseteq L}\widetilde{W}(S)$.
3. Let $\widetilde{W}(S \cup \{i\}) \leq \widetilde{W}(S)$ for some $S,\ i$. Then $\forall R \supset S\ \ \widetilde{W}(R \cup \{i\}) \leq \widetilde{W}(R)$.

The dual properties to the ones above determine optimization algorithms for the case of a supermodular function.

Our purpose is to find out under what conditions the social welfare function is submodular, supermodular or meets some similar properties that permit to employ efficient algorithms for its maximization.

In the present paper we consider only networks with *tree-type* graphs, where there are no cycles, loops or multiple edges (see [4] for the study of some network markets with parallel edges or chains). Our study shows that the desirable properties of the welfare function for a network market closely relate to the flow structure and its invariance under any increment of transmission capacities. The following examples show that in general the function is neither submodular nor supermodular even for chain-type graphs.

Consider a market with 3 nodes and flow directions that correspond to Fig.1a under any $\vec{Q}$. Then the function is supermodular according to Theorem 3.1 given below. If flow directions meet Fig.1b, then the function is submodular by Theorem 3.3. In general a chain-type market may include both structures as its components and meet none of the conditions of super- or submodularity. Moreover, flow directions may change as the capacities increase. Below we establish conditions of the flow structure conservation for chain-type and star-type network markets.

Denote $\tilde{W}(\bar{L})$ the maximum social welfare of the problem in Eq. (3), where the transmission capacities are expanded only for lines $l \in \bar{L}$.

Fig. 1. Flow structures that determine (a) supermodular and (b) submodular welfare functions.

Theorem 3.1. *For a chain-type market with n nodes, let the initial prices $p_i(\overrightarrow{Q^0})$, $i = 1, 2, \ldots, n$, monotonously decrease (as in Fig. 2). Then, for any $\overrightarrow{Q} \geq \overrightarrow{Q^0}$, $p_i(\overrightarrow{Q}) \geq p_{i+1}(\overrightarrow{Q})$, $i = 1, 2, \ldots, n-1$, and function $\widetilde{W}(\overline{L})$ is supermodular. The complexity of search for the optimal set $\overline{L}^*$ under $\overrightarrow{Q^0} = 0$ does not exceed $\frac{(n-1)n}{2}$.*

Proof. It suffices to show that the increment $\widetilde{W}(L \cup i) - \widetilde{W}(L)$ is monotonous in the set L, where $i \notin L$. Let $W(Q_i, L)$ denote the optimal welfare under given Q_i and the set L of expanded lines. Then

$$\frac{dW(Q_i, L)}{dQ_i} = p_i(Q_i, L) - p_{i+1}(Q_i, L) - e_v'^i(Q_i - Q_i^0), \tag{4}$$

where $p_i(Q_i, L)$ is a competitive equilibrium price at node i for the market (see [4] for the proof for a two-node market). So the increment may be represented as follows:

$$\widetilde{W}(L \cup i) - \widetilde{W}(L) = \int_{Q_i^0}^{Q_i^*(L)} \frac{dW(Q_i, L)}{dQ_i} dQ_i = \int_{Q_i^0}^{Q_i^*(L)} (p_i(Q_i, L) - p_{i+1}(Q_i, L) - e_v'^i(Q_i - Q_i^0))dQ_i, \tag{5}$$

where $Q_i^*(L)$ meets the equation $p_i(Q_i^*(L), L) - p_{i+1}(Q_i^*(L), L) = e_v'^i(Q_i^*(L) - Q_i^0)$. Under fixed Q_i, $p_i(Q_i, L)$ increases in L, $p_{i+1}(Q_i, L)$ decreases in L, so the increment does not decrease in L.

The following algorithm may be used to find the optimal set L^* under $\overrightarrow{Q^0} = 0$. For each $l = 1, 2, \ldots, n-1$, we check whether $\widetilde{W}(l) > \widetilde{W}(\emptyset)$. Each such line is included in $\overline{L}^*$. Then we search for the neighboring pairs that provide an additional increment of the welfare, then for neighboring triplets, and so on. The upper bound of $\overline{L}^*$ search complexity is $\frac{(n-1)n}{2}$. □

Consider a star-type market with $n + 1$ nodes such that, under initial transmission capacity $\overrightarrow{Q^0}$, 0 is a transit node, $I_1 = \{1, 2, \ldots, m\}$ is a set of producing nodes, $I_2 = \{m+1, \ldots, n\}$ is a set of consuming nodes (Fig. 3).

Theorem 3.2. *For a star-type market, the flow directions conservation holds if and only if $p_i(\overrightarrow{Q^0} \| \bar{Q}_{I_1}) \geq p_0(\overrightarrow{Q^0} \| \bar{Q}_{I_1})$, $\forall i \in I_1$, and $p_i(\overrightarrow{Q^0} \| \bar{Q}_{I_2}) \leq p_0(\overrightarrow{Q^0} \| \bar{Q}_{I_2})$, $\forall i \in I_2$, where $\bar{Q}_i = \infty$, $i \in I$.*

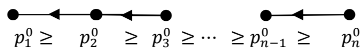

Fig. 2. Chain-type market

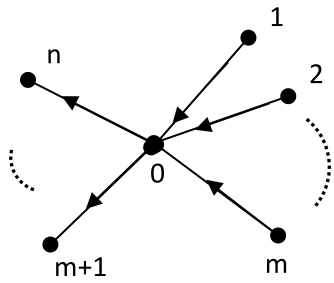

Fig. 3. Star-type market

Proof. The necessity is straightforward, since the infinite capacity increment is a particular case.

The logic for sufficiency proof is as follows. If a transmission capacities increase for parallel lines connecting producers and the transit node does not imply flow inversion along any of these parallel lines, then the price in the transit node 0 decreases monotonously. Thus, if under the maximal transmission capacities the price p_0 in the transit node 0 remains greater or equal to the price in node i, then for all intermediate values of transmission capacities this is also true. The proof for consuming nodes is similar. □

Theorem 3.3. *Consider a star-type market that meets the conditions of flow directions conservation. The social welfare function $\widetilde{W}(L_1 \cup L_2)$, where $L_1 \subseteq I_1$, $L_2 \subseteq I_2$, is submodular in L_1 under fixed set L_2; and is also submodular in L_2 under fixed set L_1.*

For any $L_1 \subseteq I_1$, $l \in I_1 \setminus L_1$, the social welfare function increment $\widetilde{W}(l \cup L_1, L_2) - \widetilde{W}(L_1, L_2)$ monotonously increases in the set L_2, and for any $L_2 \subseteq I_2$, $l \in I_2 \setminus L_2$, the social welfare function increment $\widetilde{W}(L_1, l \cup L_2) - \widetilde{W}(L_1, L_2)$ monotonously increases in the set L_1.

Proof. The monotonous increase of the social welfare function in the expanded set of consumers connecting lines follows from the equilibrium condition in the transit node. The expansion of consumers connecting lines leads to the increase of the demand in the transit node and, thus, the welfare increase.

To address submodularity, it suffices to check that, for any $l \in I_1$, $L, L' \subset I_1$,

$$\widetilde{W}(L \cup \{l\}) - \widetilde{W}(L) \geq \widetilde{W}(L \cup L' \cup \{l\}) - \widetilde{W}(L \cup L')$$

we consider the components of the welfare function. For fixed costs, the relation holds as an equality. For the case, where the variable expansion costs are zero, $\widetilde{W}(L) = \int \min\{S(p, L), D(p)\}dp - \sum_{l \in L} e_f^l$, $L \subseteq I_1$, $S(p, L) = \sum_{l \in L} S_l(p)$ the total welfare increment is the less, the greater the expanded set (see Fig. 4).

In a general case we can compute and compare the social welfare in a similar way if we shift the supply and demand functions to the central node 0. They will change according to the marginal costs $e_v'^l(\triangle Q_l)$. In particular, the supply function $S_l(p_0)$ for node $l \in I_1$ is found from the condition: $p_0 = c_l'(Q_l) + e_t^l + e_v'^l(\triangle Q_l)$. The rest arguments are the same, since $S_l(p)$ remain monotonous, the equilibrium price is the point of intersection of the demand and the total supply curves, and the social welfare $\widetilde{W}(\bar{L})$ can be found as the integral.

□

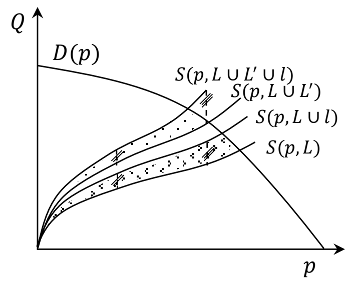

Fig. 4. The welfare $\widetilde{W}(\bar{L})$ increments under adding line l to the set L and to the set $L \cup L'$ in a star-type market

Chain-type and star-type graphs are special cases of tree-type networks (see Fig. 5). The obtained above results may be generalized for this case as follows.

Theorem 3.4. *For a tree-type market, let producers and consumers be located in different branches, that is the flow directions are from end nodes to the root in producing branches and from the root to end nodes in consuming branches of the graph, see Fig. 5, and the directions do not change under any increase of transmission capacities. Let* $\overline{L}$ *be a set of consecutive edges. Then the social welfare function* $\widetilde{W}(L_1, L_2)$*, where* $L_1 \subseteq \overline{L}$*,* $L_2 \subseteq L \setminus \overline{L}$*, is supermodular by* L_1 *in* $\overline{L}$*. If* $\overline{L}$ *is a set of parallel edges, then function* $\widetilde{W}(L_1, L_2)$ *is submodular by* L_1 *in* $\overline{L}$*.*

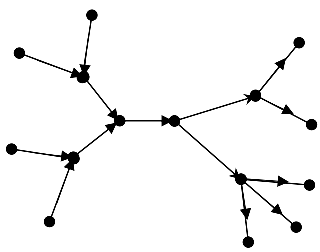

Fig. 5. Tree-type market

These properties of tree-type markets allow efficient use of algorithms of submodular and supermodular functions maximization in order to solve the transmission expansion problem.

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